* **Match the following.** [5]

1.

Part (a)	Part (b)
1. Value of $(1 + i)(1 + i^2)(1 + i^3)(1 + i^4)$	(a) 0
2. $a = 1 + i$ then value of a^2	(b) $-i$
3. Square root of $-i$	(c) $2i$
4. i^{135}	(d) $\pm\frac{1}{\sqrt{2}}(1 - i)$
5. i^{-999}	(e) i

Ans. : (1-a), (2-c), (3-d), (4-b), (5-e)

* **Choose the right answer from the given options. [1 Marks Each]** [52]

2. If $i^2 = -1$, then the sum $i + i^2 + i^3 + ...$ upto 1000 terms is equal to:

(A) 1 (B) -1 (C) i (D) 0

Ans. :

d. 0

Solution:

$\mathbf{i + i^2 + i^3 + i^4 ... i^{1000}}$

$\mathbf{i + i^2 + i^3 + i^4\ [\because i^2 = -1, i^3 = -i \text{ and } i^4 = 1]}$

$\mathbf{= i - 1 - i + 1}$

$\mathbf{= 0}$

Similarly, the sum of the next four terms of the series will be equal to 0. This is because the powers of i follow a cyclicity of 4. Hence, the sum of all terms, till 1000, will be zero.

$\mathbf{i + i^2 + i^3 + i^4 ... i^{1000} = 0}$

3. If, $x^4 + 4x^3 + 6ax^2 + 6bx + c$ is divisible by $x^3 + 3x^2 + 9x + 3$. Then, what is the value of a + b + c ?

(A) 4 (B) 6 (C) 7 (D) 10

Ans. :

c. 7

Solution:

Here, f(x) $= x^4 + 4x^3 + 6ax^2 + 6bx + c$ so, let its roots be, $\alpha, \beta, \gamma, \delta$ and

g(x) $= x^3 + 3x^2 + 9x + 3$ so, let its roots be, α, β, γ

So, from here we can conclude,

$\alpha + \beta + \gamma + \delta = -4$ and $\alpha + \beta + \gamma + \delta = -3$

Thus, $\delta = -1$

This means, $(x + 1)(x^3 + 3x^2 + 9x + 3)$

On solving this equation in simpler form we get,

$x^4 + 4x^3 + 12x^2 + 12x + 3$

$\Rightarrow 6a = 12 \Rightarrow a = 2$

$\Rightarrow 6b = 12 \Rightarrow b = 2$

$\Rightarrow c = 3$

$\Rightarrow a + b + c = 7$

4. If p and q are the roots of the equation $x^2 + px + q = 0$ then, what are the values of p and q?

(A) p = 1, q = -2 (B) p = 0, q = 1 (C) p = -2, q = 0 (D) p = -2, q = 1

Ans. :

a. p = 1, q = -2

Solution:

Since, p and q are the roots of the equation $x^2 + px + q = 0$

So, p + q = -p and pq = q

So, pq = q

And, q = 0 or p = 1

If, q = 0 then, p = 0 and if p = 1 then q = -2.

5. A real value of *x* satisfies the equation $\frac{3-4ix}{3+4ix} = a - ib (a, b \in R)$, if $a^2 + b^2 =$

(A) 1 (B) -1 (C) 2 (D) -2

Ans. :

a. 1

Solution:

$$a-ib = \frac{3-4ix}{3+4ix}$$
$$= \frac{3-4ix}{3+4ix} \times \frac{3-4ix}{3-4ix}$$
$$= \frac{9+16x^2i^2-24xi}{9-16x^2i^2}$$
$$= \frac{(9-16x^2)-i(24x)}{9+16x^2}$$
$$\Rightarrow |a-ib|^2 = \left|\frac{(9-16x^2)-i(24x)}{9+16x^2}\right|^2$$
$$\Rightarrow a^2+b^2 = \frac{(9-16x^2)^2+(24x)^2}{(9+16x^2)^2}$$
$$= \frac{81+256x^4-288x^2+576x^2}{(9+16x^2)^2}$$
$$= \frac{81+256x^4+288x^2}{(9+16x^2)^2}$$
$$= \frac{(9+16x^2)^2}{(9+16x^2)^2}$$
$$= 1$$

6. If $x^2 + px + 1 = 0$ and $(a - b)x^2 + (b - c)x + (c - a) = 0$ have both roots common, then what is the form of a, b, c?

(A) a, b, c are in A.P (B) b, a, c are in A.P (C) b, a, c are in G.P (D) b, a, c are in H.P

Ans. :

b. b, a, c are in A.P

Solution:

Given, $(a - b)x^2 + (b - c)x + (c - a) = 0$ and $x^2 + px + 1 = 0$

So, $\frac{1}{(a-b)} = \frac{p}{(b-c)} = \frac{1}{(c-a)}$

Equating the above equation, we get,

(b – c) = p(a – b) and

(b – c) = p(c – a)

So, p(a – b) = p(c – a)

⇒ a – b = c – a

So, 2a = b + c which means that b, a, c are in A.P.

7. If $i^2 = -1$, then the sum $i + i^2 + i^3 + \ldots$ upto 1000 terms is equal to:

(A) 1 (B) -1 (C) i (D) 0

Ans. :

d. 0

Solution:

$i + i^2 + i^3 + i^4 + \ldots i^{1000}$

$i + i^2 + i^3 + i^4$ [$\because i^2 = -1, i^3 = -i$ and $i^4 = 1$]

$= i - 1 - i + 1$

$= 0$

Similarly, the sum of the next four terms of the series will be equal to 0. This is because the powers of i follow a cyclicity of 4. Hence, the sum of all terms, till 1000, will be zero.

$i + i^2 + i^3 + i^4 + \ldots i^{1000} = 0$

8. According to De Moivre's theorem what is the value of $z^{\frac{1}{n}}$

(A) $r^{\frac{1}{n}}[\cos 2kn + \theta) + i\sin(2kn + \theta)]$

(B) $r^{\frac{1}{n}}\left[\frac{\cos 2kn + \theta)}{n} - \frac{i\sin\ (2kn + \theta)}{n}\right]$

(C) $r^{\frac{1}{n}}\left[\frac{\cos 2kn + \theta)}{n} + \frac{i\sin\ (2kn + \theta)}{n}\right]$

(D) $r^{\frac{1}{n}}[\cos 2kn + \theta) - i\sin(2kn + \theta)]$

Ans. :

c. $r^{\frac{1}{n}}[\frac{\cos 2kn + \theta)}{n} + \frac{i\sin\ (2kn + \theta)}{n}]$

Solution:

If n is any integer, then $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

Writing the binomial expansion of $(\cos\theta + i\sin\theta)^n$ and equating real parts of $\cos(n\theta)$ and the imaginary part to $\sin(n\theta)$ we ge

$\cos n\theta = \cos^n\theta - nc_2\cos^n - 2\theta\sin^2\theta + nc_4\cos^{n\text{-}4}\theta\sin^4\theta + \ldots \sin(n\theta) = nc_1\cos^{n\text{-}1}\theta\sin\theta - nc_3\cos^{n\text{-}3}\theta\sin^3\theta + \ldots$

If, n is a rational number, then one of the value of $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

If, $n = \frac{p}{q}$, where, p and q are integers $(q > \theta)$ and p, q have no common factor, then $(\cos\theta + i\sin\theta)^n$ has q distinct values one of which is $(\cos\theta + i\sin\theta)^n$

If, $z^{\frac{1}{n}}\left[\frac{\cos\ (2k\pi + \theta)}{n}\right] + \left[\frac{i\sin\ (2k\pi + \theta)}{n}\right]$where k = 0, 1, 2, ..., n – 1.

9. If $\frac{1 - ix}{1 + ix} = a + ib$, then $a^2 + b^2 =$

(A) 1 (B) -1 (C) 0 (D) none of these

Ans. :

a. 1

Solution:

$\frac{1 - ix}{1 + ix} = a + ib$

Taking modulus on both the sides, we get:

$\left|\frac{1 - ix}{1 + ix}\right| = |a + ib|$

$\Rightarrow \frac{\sqrt{1^2 + x^2}}{\sqrt{1^2 + x^2}} = \sqrt{a^2 + b^2}$

$\Rightarrow \sqrt{a^2 + b^2} = 1$

Squaring both the sides, we get:

$a^2 + b^2 = 1$

10. Find mirror image of point representing x + i y on real axis:

(A) (x, y) (B) (-x, -y) (C) (-x, y) (D) (x, -y)

Ans. :

d. (x, -y)

Solution:

Mirror image of point (x, y) on real axis is (x, -y).

Since real axis is acting as mirror x-coordinate remains same whereas y-coordinate gets inverted.

So, (x, -y) is mirror image of (x, y) on real axis.

11. If $(1+i)(1+2i)(1+3i)...(1+ni) = a+ib$, then 2.5.10.17...$(1+n^2)$ =

(A) $a-ib$ (B) a^2-b^2 (C) a^2+b^2 (D) none of these

Ans. :

c. a^2+b^2

Solution:

$(1+i)(1+2i)(1+3i)...(1+ni) = a+ib$

Taking modulus on both the sides, we get,

$|(1+i)(1+2i)(1+3i)...(1+ni)| = a+ib$

$|(1+i)(1+2i)(1+3i)...(1+ni)|$ can be wriiten as $|(1+i)||(1+2i)||(1+3i)|...|(1+ni)|$

$\therefore \sqrt{1^2+1^2} \times \sqrt{1^2+2^2} \times \sqrt{1^2+3^2}... \times \sqrt{1^2+n^2} = \sqrt{a^2+b^2}$

$\therefore \sqrt{2} \times \sqrt{5} \times \sqrt{10}... \times \sqrt{1^2+n^2} = \sqrt{a^2+b^2}$

Squaring on both the sides, we get:

$2 \times 5 \times 10... \times (1+n^2) = \sqrt{a^2+b^2}$

12. The value of $(1+i)(1+i^2)(1+i^3)(1+i^4)$ is

(A) 2 (B) 0 (C) 1 (D) i

Ans. :

b. 0

Solution:

$\mathbf{(1+i)(1+i^2)(1+i^3)(1+i^4)}$

$\mathbf{= (1+i)(1-1)(1-i)(1+1)\ (\because i^2 = -1, \beta = -i \text{ and } i^4 = 1)}$

$\mathbf{= (1+i)(0)(1-i)(2)}$

$\mathbf{= 0}$

13. The square root of (- 15 – 8i) is:

(A) $\pm(1-4i)$ (B) $\pm(1+4i)$ (C) $\pm(-2+4i)$ (D) None of these

Ans. :

a. $\pm(1-4i)$

14. If $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is a real number and $0 < \theta < 2\pi$, then $\theta =$

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Ans. :

a. π

Solution:

Given:

$\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is a real number

On rationalising, we get,

$\frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$

$$= \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{(1)^2-(2i\sin\theta)^2}$$
$$= \frac{3+2i\sin\theta+6i\sin\theta+4i^2\sin^2\theta}{1+4\sin^2\theta}$$
$$= \frac{3-4i\sin^2\theta+8i\sin\theta}{1+4\sin^2\theta}\ [\because i^2 = -1]$$
$$= \frac{3-4i\sin^2\theta}{1+4\sin^2\theta} + i\frac{8\sin\theta}{1+4\sin^2\theta}$$

For the above term to be real, the imaginary part has to be zero.

$\therefore \frac{8\sin\theta}{1+4\sin^2\theta} = 0$

$\Rightarrow 8\sin\theta = 0$

For this to be zero,

$\sin\theta = 0$

$\Rightarrow \theta = 0, \pi, 2\pi, 3\pi...$

But $0 < \theta < 2\pi$

Hence, $\theta = \pi$

15. If, α and β are the roots of the equation $2x^2 - 3x - 6 = 0$, then what is the equation whose roots are $a^2 + 2$ and $\beta^2 + 2$

(A) $4x^2 + 49x + 118 = 0$ (B) $4x^2 - 49x + 118 = 0$ (C) $4x^2 - 49x - 118 = 0$ (D) $x^2 - 49x + 118 = 0$

Ans. :

b. $4x^2 - 49x + 118 = 0$

Solution:

Let, $y = x^2 + 2$

Then, $2x^2 - 3x - 6 = 0$

So, $(3x)^2 = (2x^2 - 6)^2$

$[2(y - 2) - 6]^2 = 9(y - 2)$

$= 4x^2 - 49x + 118 = 0$

16. If $x+iy = (1 + i)(1 + 2i)(1 + 3i)$, then $x^2 + y^2 =$

(A) 0 (B) 1 (C) 100 (D) none of these

Ans. :

c. 100

Solution:

$\therefore x + iy = (1 + i)(1 + 2i)(1 + 3i)$

Taking modulus on both the sides:

$|x + iy| = |(1 + i)(1 + 2i)(1 + 3i)|$

$\Rightarrow |x + iy| = |(1 + i)| \times |(1 + 2i)| \times |(1 + 3i)|$

$\Rightarrow \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2}\sqrt{1^2 + 2^2}\sqrt{1^2 + 3^2}$

$\Rightarrow \sqrt{x^2 + y^2} = \sqrt{2}\sqrt{5}\sqrt{10}$

$\Rightarrow \sqrt{x^2 + y^2} = \sqrt{100}$

Squaring both the sides,

$\Rightarrow x^2 + y^2 = 100$

17. If, $(a + 1)x^2 + 2(a + 1)x + (a - 2) = 0$ then, for what parameter of ‘a’ the given equation have real and distinct roots?

(A) $(-\infty, \infty)$ (B) $(-1, \infty)$ (C) $[-1, \infty)$ (D) $(-1, 1)$

Ans. :

b. $(-1, \infty)$

Solution:

For, real and distinct roots, $D > 0$

Where, $D = b^2 - 4ac$

In the equation, $(a+1)x^2 + 2(a+1)x + (a-2) = 0$

$D = \left[2(a+1)\right]^2 - 4(a+1)(a-2)$

$= 4a^2 + 4 + 8a - 4(a^2 - 2a + a - 2)$

$= 4a^2 + 4 + 8a - 4a^2 + 4a + 8 > 0$

$\Rightarrow 12a + 12 > 0$

$\Rightarrow 12a > -12$

$\Rightarrow a > -1$

$\therefore a \in (-1, \infty)$

18. If $z = 1 - \cos\theta + i\sin\theta$, then $|z| =$

(A) $2\sin\frac{\theta}{2}$ (B) $2\cos\frac{\theta}{2}$ (C) $2\left|\sin\frac{\theta}{2}\right|$ (D) $2\left|\cos\frac{\theta}{2}\right|$

Ans. :

c. $2\left|\sin\frac{\theta}{2}\right|$

Solution:

$\therefore z = 1 - \cos\theta + i\sin\theta$

$\Rightarrow |z| = \sqrt{(1-\cos\theta)^2 + \sin^2\theta}$

$\Rightarrow |z| = \sqrt{1 + \cos^2\theta - 2\cos\theta + \sin^2\theta}$

$\Rightarrow |z| = \sqrt{1 + 1 - 2\cos\theta}$

$\Rightarrow |z| = \sqrt{2(1 - 2\cos\theta)}$

$\Rightarrow |z| = \sqrt{4\sin^2\frac{\theta}{2}}$

$\Rightarrow |z| = 2\left|\sin\frac{\theta}{2}\right|$

19. If one root of the equation $x^2 + px + 12 = 0$, is 4, while the equation $x^2 + px + q = 0$ has equal roots, the value of q is:

(A) $\frac{49}{4}$ (B) $\frac{4}{49}$ (C) 4 (D) None of these.

Ans. :

a. $\frac{49}{4}$

Solution:

It is given that, 4 is the root of the equation $x^2 + px + 12 = 0$.

$\therefore$ $16 + 4p + 12 = 0$

$\Rightarrow p = -7$

It is also given that, the equation $x^2 + px + q = 0$ has equal roots. So, the discriminant of:

$x^2 + px + q = 0$ will be zero.

$\therefore$ $p^2 - 4q = 0$

$\Rightarrow 4q = (-7)^2 = 49$

$\Rightarrow q = \frac{49}{4}$

20. If $z = 2 - 3i$ then $z^2 - 4z + 13 =$

(A) 0 (B) 1 (C) 2 (D) 3

Ans. :

a. 0

Solutions:

z = 2 - 3i

$z^2 = 2^2 - 3^2 - 12i$

= -5 - 12i

$\therefore$ $z^2 - 4z + 13$

= (-5 - 12i) - 4(2 - 3i) + 13

= -5 - 12i - 8 + 12i + 13

= -13 + 13

= 0

21. The complex number z which satisfies the condition $\left|\frac{i+z}{i-z}\right| = 1$ lies on:

(A) Circle $x^2 + y^2 = 1$ (B) The x-axis (C) The y-axis (D) The line x + y = 1

Ans. :

b. The x-axis

Solution:

$$\left|\frac{i+z}{i-z}\right| = 1$$

$$\Rightarrow \left|\frac{i+z}{i-z}\right|^2 = 1^2$$

$$\Rightarrow \boldsymbol{\left(\frac{i+z}{i-z}\right)\overline{\left(\frac{i+z}{i-z}\right)} = 1}$$

$$\Rightarrow \left(\frac{i+z}{i-z}\right)\left(\frac{-i+\bar{z}}{-i-\bar{z}}\right) = 1$$

$$\Rightarrow \left(\frac{-i^2 - zi + \bar{z}i + z\bar{z}}{-i^2 + zi - \bar{z}i + z\bar{z}}\right) = 1$$

$$\Rightarrow -i^2 - zi + \bar{z}i + z\bar{z} = -i^2 + zi - \bar{z}i + z\bar{z}$$

$$\Rightarrow -zi + \bar{z}i = zi - \bar{z}i$$

$$\Rightarrow \bar{z}i + \bar{z}i = zi - zi$$

$$\Rightarrow 2\bar{z}i = 2zi$$

$$\Rightarrow \bar{z} = z$$

⇒ z is purely real.

22. If $a = \cos\theta + i\sin\theta,$ then $\frac{1+a}{1-a} =$

(A) $\cot\frac{\theta}{2}$ (B) $\cot\theta$ (C) $i\cot\frac{\theta}{2}$ (D) $i\tan\frac{\theta}{2}$

Ans. :

c. $i\cot\frac{\theta}{2}$

Solution:

$a = \cos\theta + i\sin\theta$ (given)

$$\Rightarrow \frac{1+a}{1-a} = \frac{1+\cos\theta + i\sin\theta}{1-\cos\theta - i\sin\theta}$$

$$\Rightarrow \frac{1+a}{1-a} = \frac{1+\cos\theta + i\sin\theta}{1-\cos\theta - i\sin\theta} \times \frac{1-\cos\theta + i\sin\theta}{1-\cos\theta + i\sin\theta}$$

$$\Rightarrow \frac{1+a}{1-a} = \frac{(1+i\sin\theta)^2 - \cos^2\theta}{(1-\cos\theta)^2 - (i\sin\theta)^2}$$

$$\Rightarrow \frac{1+a}{1-a} = \frac{1 - i\sin^2\theta + 2i\sin\theta - \cos^2\theta}{1+\cos^2\theta - 2\cos\theta + \sin^2\theta}$$

$$\Rightarrow \frac{1+a}{1-a} = \frac{1 - (\sin^2\theta + \cos^2\theta) + 2i\sin\theta}{1 + (\sin^2\theta + \cos^2\theta) - 2\cos\theta}$$

$$\Rightarrow \frac{1+a}{1-a} = \frac{2i\sin\theta}{2(1-\cos\theta)}$$

$$\Rightarrow \frac{1+a}{1-a} = \frac{2i\sin\frac{\theta}{2} - \cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$

$$\Rightarrow \frac{1+a}{1-a} = \frac{i\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$

$$\Rightarrow \frac{1+a}{1-a} = i\cot\frac{\theta}{2}$$

23. The argument of $\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$ is:

(A) 60° (B) 120° (C) 210° (D) 240°

Ans. :

d. 240°

Solution:

$$\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$$

Rationalising the denominator,

$$\frac{1-i\sqrt{3}}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$= \frac{1+3i^2-2\sqrt{3}i}{1-3i^2}$$

$$= \frac{-2-2\sqrt{3}i}{4} \quad (\because i^2 = -1)$$

$$= \frac{-1}{2} - i\frac{\sqrt{3}}{2}$$

Then, $\Rightarrow \tan\alpha = \left|\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right|$

$= \sqrt{3}$

$\Rightarrow \alpha = 60^\circ$

Since the points $(\frac{-1}{2}, -\frac{-\sqrt{3}}{2})$ lie in the third quadrant, the argument is given by:

$\theta = 180^\circ + 60^\circ$

$= 240^\circ$

24. If, $(a+1)x^2 + 2(a+1)x + a - 2 = 0$ then, for what parameter of 'a' the given equation have equal roots?

(A) $(-\infty, -1)$ (B) $[-1, \infty)$ (C) $(0, 1)$ (D) Not possible

Ans. :

d. Not possible

Solution:

For, equal roots, $D = 0$

Where, $D = b^2 - 4ac$

In the equation, $(a+1)x^2 + 2(a+1)x + a - 2 = 0$

$D = \left[2(a+1)\right] - 4(a+1)(a-2)$

$= 4a^2 + 4 + 8a - 4(a^2 - 2a + a - 2)$

$= 4a^2 + 4 + 8a - 4a^2 + 4a + 8 > 0$

$\Rightarrow 12a + 12 = 0$

$\Rightarrow 12a = -12$

$\Rightarrow a = -1$

So, from here it is clear that $a = -1$ is not possible because the equation is becoming linear.

25. Choose the correct answer.
$\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other for:
(A) $x = n\pi$ (B) $x = \left(n + \frac{1}{2}\right)\frac{\pi}{2}$ (C) $x = 0$ (D) no value of x

Ans. :
d. no value of x.
Solution:
$\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other.
$\Rightarrow \overline{\sin x + i\cos 2x} = \cos x - i\sin 2x$
$\Rightarrow \sin x - i\cos 2x = \cos x - i\sin 2x$
On comparing real and imaginary parts of both the sides, we get
$\sin x = \cos x$ and $\cos 2x = \sin 2x$
$\Rightarrow \tan x = 1$ and $\tan 2x = 1$
Now, $\tan 2x = 1$
$\Rightarrow \frac{2\tan x}{1 - \tan^2 x} = 1,$ which is not satisfied by $\tan x = 1$
Hence, no value of x is possible.

26. Convert -1 + i into polar form:
(A) $\sqrt{2}, \frac{5\pi}{4}$ (B) $\sqrt{2}, \frac{3\pi}{4}$ (C) $-\sqrt{2}, \frac{\pi}{4}$ (D) $\sqrt{2}, \frac{\pi}{4}$

Ans. :
b. $\sqrt{2}, \frac{3\pi}{4}$
Solution:

$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2}$
$r\cos\theta = -1$ and $r\sin\theta = 1$ so, θ is in 2nd quadrant since sin is positive and cos is negative.
$\tan\theta = -1 \Rightarrow \tan\theta = \frac{-\tan\pi}{4}$
$\Rightarrow \tan\theta = \tan(\frac{\pi - \pi}{4}) = \frac{\tan 3\pi}{4}$
$\Rightarrow \theta = \frac{3\pi}{4}$

27. Choose the correct answer.
The value of $(z + 3)(\bar{z} + 3)$ is equivalent to:
(A) $|z + 3|^2$ (B) $|z - 3|$ (C) $z^2 + 3$ (D) None of these.

Ans. :
a. $|z + 3|^2$
Solution:
Let z = x + iy. Then
$(z + 3)(\bar{z} + 3) = (x + iy + 3)(x - iy + 3)$
$= (x + 3)^2 - (iy)^2$
$= (x + 3)^2 + y^2$
$= |x + 3 + iy|^2 = |z + 3|^2$

28. A quadratic equation $ax^2 + bx + c = 0$ has two distinct real roots, if
(A) a = 0 (B) $b^2 - 4ac = 0$ (C) $b^2 - 4ac < 0$ (D) $b^2 - 4ac > 0$

Ans. :

d. $b^2 - 4ac > 0$

Solutions:

If a = 0, it becomes linear equation.

If $b^2 - 4ac = 0$, then there will be real and equal roots.

If $b^2 - 4ac < 0$, then the roots will be unreal.

Only if $b^2 - 4ac > 0$, we will get two real distinct roots.

Option D is correct!

29. The number of real roots of the equation $(x^2 + 2x)^2 - (x + 1)^2 - 55 = 0$:

(A) 2 (B) 1 (C) 4 (D) None of these.

Ans. :

a. 2

Solution:

$(x^2 + 2x)^2 - (x + 1)^2 - 55 = 0$

$\Rightarrow (x^2 + 2x + 1 - 1)^2 - (x + 1)^2 - 55 = 0$

$\Rightarrow \left\{(x+1)^2 - 1\right\}^2 - (x+1)^2 - 55 = 0$

$\Rightarrow \left\{(x+1)^2\right\}^2 + 1 - 3(x+1)^2 - 55 = 0$

$\Rightarrow \left\{(x+1)^2\right\}^2 - 3(x+1)^2 - 54 = 0$

Let $p = (x + 1)^2$

$\Rightarrow p^2 - 3p - 54 = 0$

$\Rightarrow p^2 - 9p + 6p - 54 = 0$

$\Rightarrow (p + 6)(p - 9) = 0$

$\Rightarrow p = 9$ or $p = -6$

Rejecting $p = -6$

$\Rightarrow (x + 1)^2 = 9$

$\Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow (x + 4)(x - 2) = 0$

$\Rightarrow x = 2, x = -4$

30. The polar form of $(i^{25})^3$ is:

(A) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (B) $\cos\pi + i\sin\pi$ (C) $\cos\pi - i\sin\pi$ (D) $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

Ans. :

d. $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

Solution:

$(i^{25})^3 = (i)^{75}$

$= (i)^{4 \times 18 + 3}$

$= (i)^3$

$= -i \; (\because i^4 = 1)$

Let $z = 0 - i$

Since, the point (0,−1) lies on the negative direction of imaginary axis.Therefor

e, $\arg(z) = \frac{-\pi}{2}$

Modulus, $r = |z| = |1| = 1$

$\therefore$ Polar form $= r(\cos\theta + i\sin\theta)$

$= \cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)$

$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

31. What will be the sum of b + c if the equations $x^2 + bx + c = 0$ and $x^2 + 3x + 3 = 0$ have one common root:

(A) 2 (B) 4 (C) 6 (D) 8

Ans. :

c. 6

Solution:

Comparing the coefficients of the above equation we get,

$\frac{1}{1} = \frac{b}{3} = \frac{c}{3}$

This means $b = 3$ and $c = 3$

$\therefore b + c = 6$

32. Choose the correct answer.

The real value of α for which the expression $\dfrac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely real is:

(A) $(n + 1)\frac{\pi}{2}$ (B) $(2n + 1)\frac{\pi}{2}$

(C) $n\pi$ (D) None of these, where $n \in N$

Ans. :

c. $z = \dfrac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$

Solution:

$= \frac{(1 - i\sin\alpha)(1 - 2i\sin\alpha)}{(1 + 2i\sin\alpha)(1 - 2i\sin\alpha)} = \frac{1 - i\sin\alpha - 2i\sin\alpha + 2i^2\sin^2\alpha}{1 - 4i^2\sin^2\alpha}$

$= \frac{1 - 3i\sin\alpha - 2\sin^2\alpha}{1 + 4\sin^2\alpha} = \frac{1 - 2\sin^2\alpha}{1 + 4\sin^2\alpha} - \frac{3i\sin\alpha}{1 + 4\sin^2\alpha}$

It is given that z is a purely real.

$\Rightarrow \frac{-3i\sin\alpha}{1 + 4\sin^2\alpha} = 0$

$\Rightarrow -3\sin\alpha = 0$

$\Rightarrow \sin\alpha = 0$

$\Rightarrow \alpha = n\pi,\ n \in I$

33. If $z_1 = 2 + 3i$ and $z_2 = 5 + 2i$, then find sum of two complex numbers:

(A) 4 + 8i (B) 3 - i (C) 7 + 5i (D) 7 - 5i

Ans. :

c. 7 + 5i

Solution:

In addition of two complex numbers, corresponding parts of two complex numbers are added i.e. real partsof both are added and imaginary parts of both are added.

So, sum = (2 + 5) + (3 + 2)i = 7 + 5i.

34. Choose the correct answer.

The real value of θ for which the expression $\dfrac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is a real number is:

(A) $n\pi + \frac{\pi}{4}$ (B) $n\pi + (-1)^n\frac{\pi}{4}$ (C) $2n\pi \pm \frac{\pi}{2}$ (D) None of these.

Ans. :

c. $2n\pi \pm \dfrac{\pi}{2}$

Solution:

Let $z = \frac{1 + i\cos\theta}{1 - 2i\cos\theta} = \frac{1 + i\cos\theta}{1 - 2i\cos\theta} \times \frac{1 + 2i\cos\theta}{1 + 2i\cos\theta}$

$= \frac{1+2i\cos\theta+i\cos\theta+2i^2\cos^2\theta}{1-4i^2\cos^2\theta}$

$= \frac{1-3i\cos\theta-2\cos^2\theta}{1+4\cos^2\theta}$

$= \frac{1-2\cos^2\theta}{1+4\cos^2\theta} + \frac{3\cos\theta}{1+4\cos^2\theta}i$

If z is real number, then

$\frac{3\cos\theta}{1+4\cos^2\theta} = 0$

$\Rightarrow 3\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

$\therefore\ \theta = (2n+1)\frac{\pi}{2},\ n \in N$

35. If $z = \left(\frac{1+i}{1-i}\right)$, then z^4 equals:

(A) 1 (B) -1 (C) 0 (D) none of these.

Ans. :

a. 1

Solution:

Let $z = \frac{1+i}{1-i}$

Rationalising the denominator:

$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$\Rightarrow z = \frac{1+i^2+2i}{1-i^2}$

$\Rightarrow z = \frac{1-1+2i}{1+1}$

$\Rightarrow z = \frac{2i}{2}$

$\Rightarrow z = i$

$\Rightarrow z^4 = i^4$

Since $i^2 = -1$, we have:

$\Rightarrow z^4 = i^2 \times i^2$

$\Rightarrow z^4 = 1$

36. What is the number of solution(s) of the equation $|\sqrt{x-2}| + \sqrt{x(\sqrt{x-4})} + 2 = 0$

(A) 2 (B) 4

(C) No solution (D) Infinitely many solutions

Ans. :

a. 2

Solution:

We have $|\sqrt{x-2}| + \sqrt{x}(\sqrt{x-4}) + 2 = 0$

$|\sqrt{x}-2| + \sqrt{(x)^2-4\sqrt{x}} + 2 = 0$

$|\sqrt{x}-2| + |\sqrt{x}-2|^2 - 2 = 0$

$|\sqrt{x}-2| = -2,\ 1$

Thus, $\sqrt{x} - 2 = +1, -1$ or $x = 1,\ 9$

37. $\frac{1+2i+3i^2}{1-2i+3i^2}$ equals:

(A) i (B) -1 (C) -i (D) 4

Ans. :

c. -i

Solution:

Let $z = \frac{1+2i+3i^2}{1-2i+3i^2}$

$\Rightarrow z = \frac{1+2i-3}{1-2i-3}$

$\Rightarrow z = \frac{-2+2i}{-2-2i} \times \frac{-2+2i}{-2+2i}$

$\Rightarrow z = \frac{(-2+2i)^2}{(-2)^2-(2i)^2}$

$\Rightarrow z = \frac{4+4i^2-8i}{4+4}$

$\Rightarrow z = \frac{4-4-8i}{8}$

$\Rightarrow z = \frac{-8i}{8}$

$\Rightarrow z = -i$

38. The value of $(1+i)^4 + (1-i)^4$ is:

(A) 8 (B) 4 (C) -8 (D) -4

Ans. :

c. -8

Solution:

Using $a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2$

$(1+i)^4 + (1-i)^4$

$= \left((1+i)^2 + (1-i)^2\right)^2 - 2(1+i)^2(1-i)^2$

$= (1 + i^2 + 2i + 1 + i^2 - 2i)^2 - 2(1 + i^2 + 2i)(1 + i^2 - 2i)$

$= (1 - 1 + 2i + 1 - 1 - 2i)^2 - 2(1 - 1 + 2i)(1 - 1 - 2i)$

$= (0) - 2(2i)(-2i)\ (\because i^2 = -1)$

$= 8i^2$

$= -8$

39. Choose the correct answer.

A real value of x satisfies the equation $(\frac{3-4ix}{3+4ix}) = \alpha - i\beta (\alpha, \beta \in R)$ if $\alpha^2 + \beta^2 =$

(A) 1 (B) -1 (C) 2 (D) -2

Ans. :

a. 1

Solution:

Given that, $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta$

$\Rightarrow 8\left(\frac{3-4ix}{3+4ix} \times \frac{3-4ix}{3-4ix}\right) = \alpha - i\beta$

$\Rightarrow \frac{9-12ix-12ix+16i^2x^2}{9-16i^2x^2} = \alpha - i\beta$

$\Rightarrow \frac{9-24ix-16x^2}{9+16x^2} = \alpha - i\beta$

$\Rightarrow \frac{9-16x^2}{9+16x^2} - \frac{24x}{9+16x^2}i = \alpha + i\beta \ldots\ldots(i)$

$\Rightarrow \frac{9-16x^2}{9+16x^2} + \frac{24x}{9+16x^2}i = \alpha + i\beta \ldots\ldots(ii)$

Multiplying eq. (i) and (ii) we get

$(\frac{9-16x^2}{9+16x^2})^2 + (\frac{24x}{9+16x^2})^2 = \alpha^2 + \beta^2$

$\Rightarrow \frac{(9-16x^2)^2+(24x)^2}{(9+16x^2)^2} = \alpha^2 + \beta^2$

$\Rightarrow \frac{81+256x^4-288x^2+576x^2}{(9+16x^2)^2} = \alpha^2 + \beta^2$

$\Rightarrow \frac{81+256x^4+288x^2}{(9+16x^2)^2} = \alpha^2 + \beta^2$

$\Rightarrow \frac{(9+16x^2)}{(9+16x^2)^2} = \alpha^2 + \beta^2$

So, $\alpha^2 + \beta^2 = 1$

40. Solve $\sqrt{3x^2} + x + \sqrt{3} = 0$

(A) $\frac{-1 \pm i\sqrt{11}}{6\sqrt{3}}$ (B) $\frac{1 \pm i\sqrt{11}}{6\sqrt{3}}$ (C) $\frac{1 \pm \sqrt{11}}{6\sqrt{3}}$ (D) $\frac{-1 \pm \sqrt{11}}{6\sqrt{3}}$

Ans. :

a. $\frac{-1 \pm i\sqrt{11}}{6\sqrt{3}}$

Solution:

$\sqrt{3x^2} + x + \sqrt{3} = 0$

$\Rightarrow 3x^2 + \sqrt{3x} + 3 = 0$

$\Rightarrow D = (\sqrt{3})^2 - 4.3.3 = 3 - 36 = -33$

Since $D \leq 0$, imaginary roots are there.

41. If $\frac{1+7i}{(2-i)^2}$, then:

(A) $|z| = 2$ (B) $|z| = \frac{1}{2}$ (C) $\text{amp}(z) = \frac{\pi}{4}$ (D) $\text{amp}(z) = \frac{3\pi}{4}$

Ans. :

d. $\text{amp}(z) = \frac{3\pi}{4}$

Solution:

$\mathbf{z = \frac{1+7i}{(2-i)^2}}$

$\mathbf{\Rightarrow z = \frac{1+7i}{4-1-4i} \; [\because i^2 = -1]}$

$\mathbf{\Rightarrow z = \frac{1+7i}{3-4i}}$

$\mathbf{\Rightarrow z = \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i}}$

$\mathbf{\Rightarrow z = \frac{3+4i+21i+28i^2}{9-16i^2}}$

$\mathbf{\Rightarrow z = \frac{3-28+25i}{9+16}}$

$\mathbf{\Rightarrow z = \frac{-25+25i}{25}}$

$\mathbf{\Rightarrow z = -1+i}$

$\mathbf{\Rightarrow \tan\alpha = \left|\frac{Im(z)}{Re(z)}\right|}$

$\mathbf{= 1}$

$\mathbf{\Rightarrow \alpha = \frac{\pi}{4}}$

Since, z lies in the second quadrant.

Therefore, $\text{amp}(z) = \pi - \alpha$

$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

42. If α, β are the roots of the equation $x^2 - p(x+1) - c = 0$, then $(\alpha+1)(\beta+1) =$

(A) c (B) c - 1 (C) 1 - c (D) None of these.

Ans. :

c. 1 - c

Solution:

Given equation:

$x^2 - p(x+1) - c = 0$

or $x^2 - px - p - c = 0$

Also α and β are the roots of the equation.

Sum of the roots $= \alpha + \beta = p$

Product of the roots $= \alpha\beta = -(c + p)$

Then, $(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$

$= -(c + p) + p + 1$

$= 1 - c$

$= -c - p + p + 1$

43. If $(x + iy)^{\frac{1}{3}} = a + ib$, then $\frac{x}{a} + \frac{y}{b} =$

(A) 0 (B) 1 (C) -1 (D) none of these

Ans. :

d. none of these.

Solution:

$(x + iy)^{\frac{1}{3}} = a + ib$

Cubing on both the sides, we get:

$x + iy = (a + ib)^3$

$\Rightarrow x + iy = a^3 + (ib)^3 + 3a^2bi + 3a(bi)^2$

$\Rightarrow x + iy = a^3 + i^3b^3 + 3a^2bi + 3i^2ab^2$

$\Rightarrow x + iy = a^3 - ib^3 + 3a^2bi - 3ab^2 \ (\because i^2 = -1, i^3 = -i)$

$\Rightarrow x + iy = a^3 - 3ab^2 + i(-b^3 + 3a^2b)$

$\therefore x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$

or, $\frac{x}{a} = a^2 - 3b^2$ and $\frac{y}{b} = 3a^2 - b^2$

$\Rightarrow \frac{x}{a} + \frac{y}{b} = a^2 - 3b^2 + 3a^2 - b^2$

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 4a^2 - 4b^2$

44. If $\alpha\beta$ are the roots of the equation $x^2 + px + q = 0$ then $-\frac{1}{\alpha} + \frac{1}{\beta}$ are the roots of the equation:

(A) $x^2 - px + q = 0$ (B) $x^2 + px + q = 0$ (C) $qx^2 + px + 1 = 0$ (D) $qx^2 - px + 1 = 0$

Ans. :

c. $qx^2 - px + 1 = 0$

Solution:

Given equation: $x^2 + px + q = 0$

Also, α and β are the roots of the given equation.

Then, sum of the roots $= \alpha + \beta = -p$

Product of the roots $= \alpha\beta = q$

Now, for roots $-\frac{1}{\alpha}, -\frac{1}{\beta}$, we have:

Sum of the roots $= -\frac{1}{\alpha} - \frac{1}{\beta} = -\frac{\alpha + \beta}{\alpha\beta} = -\left(\frac{-p}{q}\right) = \frac{p}{q}$

Product of the roots $= \frac{1}{\alpha\beta} = \frac{1}{q}$

Hence, the equation involving the roots $-\frac{1}{\alpha}, -\frac{1}{\beta}$ is as follows:

$x^2 + (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - \frac{p}{q}x + \frac{1}{q} = 0$

$\Rightarrow qx^2 - px + 1 = 0$

45. The number of roots of the equation $\frac{(x+2)(x-5)}{(x+3)(x+6)} = \frac{x-2}{x+4}$ is:

(A) 0 (B) 1 (C) 2 (D) 3

Ans. :

b. 1

Solution:

$\frac{(x+2)(x-5)}{(x+3)(x+6)} = \frac{x-2}{x+4}$

$\Rightarrow (x^2 - 3x - 10)(x + 4) = (x^2 + 3x - 18)(x - 2)$

$\Rightarrow x^3 + 4x^2 - 3x^2 - 12x - 10x - 40 = x^3 - 2x^2 + 3x^2 - 6x - 18x + 36$

$\Rightarrow x^2 - 22x - 40 = x^2 - 24x + 36$

$\Rightarrow 2x = 76$

$\Rightarrow x = 38$

Hence, the equation has only 1 root.

46. If α, β are the roots of the equation x2 + px + 1 = 0; γ, δ the roots of the equation $x^2 + qx + 1 = 0$, then $(\alpha - \gamma)(\alpha + \delta)(\beta - \delta)(\beta + \delta) =$

(A) $q^2 - p^2$ (B) $p^2 - q^2$ (C) $p^2 + q^2$ (D) None of these.

Ans. :

a. $q^2 - p^2$

Solution:

Given: α and β are the roots of the equation $x^2 + px + 1 = 0$.

Also, γ and $\delta\gamma$ and δ are the roots of the equation $x^2 + qx + 1 = 0$

Then, the sum and the product of the roots of the given equation are as follows:

$\alpha + \beta = -\frac{p}{1} = -p$

$\alpha\beta = \frac{1}{1} = 1$

$\gamma + \delta = -\frac{q}{1} = -P$

$\gamma\delta = \frac{1}{1} = 1$

Moreover, $(\gamma - \delta)^2 = \gamma^2 + \delta^2 + 2\gamma\delta$

$\Rightarrow \gamma^2 + \delta^2 = q^2 - 2$

$\therefore (\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta + \delta) = (\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$

$= (\alpha\beta - \alpha\gamma - \beta\gamma + \gamma^2)(\alpha\beta + \alpha\gamma + \beta\delta + \delta^2)$

$= [\alpha\beta - \gamma(\alpha + \beta) + \gamma^2][\alpha\beta + \delta(\alpha + \beta) + \delta^2])$

$= (1 - \gamma(-p) + \gamma^2)(1 + \delta(-p + \delta^2))$

$= (1 + \gamma p + \gamma^2)(1 - \delta p + \delta^2)$

$= (1 + \gamma p + \gamma^2)(1 - \delta p + \delta^2)$

$= 1 - p\delta + \delta^2 + p\gamma - p^2\gamma\delta + p\gamma\delta^2 + \gamma^2 - p\delta y^2 + \gamma^2\delta^2$

$= 1 - p\delta + p\gamma + \delta^2 - p^2\gamma\delta + p\gamma\delta^2 + \gamma^2 = p\delta\gamma^2 + \gamma^2\delta^2$

$= 1 - p(\delta - \gamma) - p^2\gamma\delta + p\gamma\delta(\delta - \gamma) + (\gamma^2 + \delta) + 1$

$= 1 - p^2 + (\delta - \gamma)p(\delta - \gamma) + (\gamma^2 + \delta^2) + 1$

$= -P^2 2 + (\delta - \gamma)p(1 - 1) + q^2$

$= q^2 - p^2$

47. The least positive integer n such that $(\frac{2i}{1+i})^n$ is a positive integer, is:

(A) 16 (B) 8 (C) 4 (D) 2

Ans. :

b. 8

Solution:

Let $z = \left(\frac{2i}{1+i}\right)$

$\Rightarrow z = \frac{2i}{1+i} \times \frac{1-i}{1-i}$

$\Rightarrow z = \frac{2i(1-i)}{1-i^2}$

$\Rightarrow z = \frac{2i(1-i)}{1+1}$ $[\because i^2 = -1]$

$\Rightarrow z = \frac{2i(1-i)}{2}$

$\Rightarrow z = i - i^2$

$\Rightarrow z = i + 1$

Now, $z^n = (1+i)^n$

For $n = 2$,

$z^2 = (1+i)^2$

$= 1 + i^2 + 2i$

$= 1 - 1 + 2i$

$= 2i \ldots (1)$

Since this is not a positive integer,

For $n = 4$,

$z^4 = (1+i)^4$

$= \left[(1+i)^2\right]^2$

$= (2i)^2$ [Using (1)]

$= (4i)^2$

$= -4 \ldots (2)$

This is a negative integer.

For $n = 8$,

$z^8 = (1+i)^8$

$= \left[(1+i)^4\right]^2$

$= (-4)^2$ [Using (2)]

$= 16$

This is a positive integer.

Thus, $z = \left(\frac{2i}{1+i}\right)^n$ is positive for $n = 8$.

Therefore, 8 is the least positive integer such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer.

48. Solve $2x^2 + \sqrt{2x+2} = 0$

(A) $\frac{-1 \pm i\sqrt{7}}{2\sqrt{2}}$ (B) $\frac{1 \pm i\sqrt{7}}{2\sqrt{2}}$ (C) $\frac{1 \pm \sqrt{7}}{2\sqrt{2}}$ (D) $\frac{-1 \pm \sqrt{7}}{2\sqrt{2}}$

Ans. :

a. $\frac{-1 \pm i\sqrt{7}}{2\sqrt{2}}$

Solution:

$2x^2 + \sqrt{2x+2} = 0$

$\Rightarrow D = (\sqrt{2})^2 - 4.2.2 = 2 - 16 = -14$

Since $D \le 0$, imaginary roots are there.

$\Rightarrow x = \frac{-\sqrt{2} \pm \sqrt{D}}{2.2} = \frac{-\sqrt{2} \pm i\sqrt{D}}{2.2} = \frac{-1 \pm i\sqrt{7}}{2\sqrt{2}}$

49. The value of $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1$ is

(A) -1 (B) -2 (C) -3 (D) -4

Ans. :

b. -2

Solution:

$$\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}-1$$

$$=\frac{i^{4\times148}+i^{4\times147+2}+i^{4\times147}+i^{4\times146+2}+i^{4\times146}}{i^{4\times145+2}+i^{4\times145}+i^{4\times144+2}+i^{4\times144}+i^{4\times143+2}}-1 \ [\because i^4=1 \text{ and } i^2=-1]$$

$$=\frac{1+i^2+1+i^2+1}{i^2+1+i^2+1+i^2}-1$$

$$=\frac{1}{-1}-1$$

$$=-2$$

50. The value of $\frac{(i^5+i^6+i^7+i^8+i^9)}{(1+i)}$ is:

(A) $\frac{1}{2}(1+i)$ (B) $\frac{1}{2}(1-i)$ (C) 1 (D) $\frac{1}{2}$

Ans. :

a. $\frac{1}{2}(1+i)$

Solution:

$$\frac{(i^5+i^6+i^7+i^8+i^9)}{(1+i)}$$

$$=\frac{i-1-i+1+i}{1+i} \ [\text{ As}, i^5=i, i^6=-1, i^7=-i, i^8=1, i^9=i]$$

$$=\frac{i}{i+1}$$

$$=\frac{i}{i+1}\times\frac{i-1}{i-1}$$

$$=\frac{i(i-1)}{i^2-1}$$

$$=\frac{i^2-i}{-2}$$

$$=\frac{1}{2}(1+i)$$

51. If $z=\cos\frac{\pi}{4}+i\sin\frac{\pi}{6}$, then

(A) $|z|=1, \arg(z)=\frac{\pi}{4}$ (B) $|z|=1, \arg(z)=\frac{\pi}{6}$

(C) $|z|=\frac{\sqrt{3}}{2}, \arg(z)=\frac{5\pi}{24}$ (D) $|z|=\frac{\sqrt{3}}{2}, \arg(z)=\tan^{-1}\frac{1}{\sqrt{2}}$

Ans. :

d. $|z|=\frac{\sqrt{3}}{2}, \arg(z)=\tan^{-1}\frac{1}{\sqrt{2}}$

Solution:

$$\mathbf{z=\cos\frac{\pi}{4}+i\sin\frac{\pi}{6}}$$

$$\Rightarrow z=\frac{1}{\sqrt{2}}+\frac{1}{2}i$$

$$\Rightarrow |z|=\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+\frac{1}{4}}$$

$$\Rightarrow |z|=\sqrt{\frac{1}{2}+\frac{1}{4}}$$

$$\Rightarrow |z|=\sqrt{\frac{3}{4}}$$

$$\Rightarrow |z|=\frac{\sqrt{3}}{2}$$

$$\Rightarrow \tan\alpha=\left|\frac{\text{Im}(z)}{\text{Re}(z)}\right|$$

$= \frac{1}{\sqrt{2}}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Since, the point z lies in the first quadrant.

Therefore, $|z| = \frac{\sqrt{3}}{2}, \arg(z) = \tan^{-1}\frac{1}{\sqrt{2}}$

52. The number of real solutions of $|2x - x^2 - 3| = 1$ is:

(A) 0 (B) 2 (C) 3 (D) 4

Ans. :

b. 2

Solution:

i. Given equation: $|2x - x^2 - 3| = 1$

$2x - x^2 - 3 = 1$

$\Rightarrow 2x - x^2 - 4 = 0$

$\Rightarrow x^2 - 2x + 4 = 0$

$\Rightarrow (x - 2)^2 = 0$

$\Rightarrow x = 2, 2$

ii. $-2x + x^2 + 3 = 1$

$\Rightarrow x^2 - 2x + 2 = 0$

$\Rightarrow x^2 - 2x + 1 + 1 = 0$

$\Rightarrow (x - 1 + i)(x - 1 - i) = 0$

$\Rightarrow x = 1 - i, 1 + i$

Hence, the real solutions are 2, 2.

53. If α and β are imaginary cube roots of unity, then the value of $\alpha^4 + \beta^{28} + \frac{1}{\alpha\beta}$ is:

(A) 1 (B) -1 (C) 0 (D) None of these

Ans. :

c. 0

*** Given section consists of questions of 2 marks each.** **[54]**

54. Express the complex number $\left(-2 - \frac{1}{3}i\right)^3$ in the form of a + ib.

Ans. : $\left(-2 - \frac{1}{3}i\right)^3 = -\left(2 + \frac{1}{3}i\right)^3$

$= -\left[(2)^3 + \left(\frac{1}{3}i\right)^3 + 3 \times (2)^2 \times \frac{1}{3}i + 3 \times 2 \times \left(\frac{1}{3}i\right)^2\right]$

$= -\left[8 + \frac{1}{27}i^3 + 4i + \frac{2}{3}i^2\right] = -\left[8 - \frac{1}{27}i + 4i - \frac{2}{3}\right]\left[\begin{array}{l}\because i^3 = -i \\ i^2 = -1\end{array}\right]$

$= \left[\left(8 - \frac{2}{3}\right) + \left(4 - \frac{1}{27}\right)i\right.$

$= -\left[\frac{22}{3} + \frac{107}{27}i\right] = \frac{-22}{3} - \frac{107}{27}i$

55. Find the multiplicative inverse of the complex numbers $= \sqrt{5} + 3i$

Ans. : M.I. of $= \sqrt{5} + 3i$

$= \frac{1}{\sqrt{5}+3i} = \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$

$= \frac{\sqrt{5}-3i}{(\sqrt{5})^2-(3i)^2}$

$= \frac{\sqrt{5}-3i}{5-9i^2} = \frac{\sqrt{5}-3i}{5+9} = \frac{1}{14}(\sqrt{5}-3i)$

56. Express the following in the form of a + ib.

$$\frac{(3+\sqrt{5}i)(3-\sqrt{5}i)}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)}$$

Ans. : We have, $\frac{(3+\sqrt{5}i)(3-\sqrt{5}i)}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)}$

$= \frac{9-3\sqrt{5}i+3\sqrt{5}i-\sqrt{5}i\sqrt{5}i}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i}$

$= \frac{9+5}{2\sqrt{2}i} = \frac{14}{2\sqrt{2}i} = \frac{7}{\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i}$

$= \frac{7\sqrt{2}i}{2i^2} = \frac{7\sqrt{2}i}{-2}$ = 0 - i $\frac{7\sqrt{2}}{2}$ = a + ib [say]

where, a = 0 and b = $\frac{-7\sqrt{2}}{2}$

57. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$.

Ans. : $\left|\frac{1+i}{1-i} - \frac{1-i}{1+i}\right| = \left|\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}\right|$

$= \left|\frac{1+i^2+2i-1-i^2+2i}{1-i^2}\right|$

$\left|\frac{4i}{2}\right| = |2i| = \sqrt{4} = 2.$

58. Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$.

Ans. : Here $|1-i|^x = 2^x$

$\Rightarrow \left[\sqrt{(1)^2+(-1)^2}\right]^x = 2^x \Rightarrow (\sqrt{2})^x = 2^x$

$\Rightarrow 2^{\frac{x}{2}} = 2^x \Rightarrow \frac{x}{2} = x \Rightarrow \frac{x}{2} - x = 0 \Rightarrow \frac{-x}{2} = 0$

$\Rightarrow x = 0$

Thus the given equation has no non-zero integral solution.

59. If (a + ib) (c + id) (e+ if) (g + ih) = A + iB then show that

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

Ans. : Here (a + ib) (c + id) (e+ if) (g + ih) = A + iB

Taking modulus on both sides

| (a + ib) (c + id) (e+ if) (g + ih) | = |A + iB|

$\Rightarrow$ |a + ib | |c + id | | e + if | |g + ih| = | A + iB|

$\Rightarrow \left(\sqrt{a^2+b^2}\right)\left(\sqrt{c^2+d^2}\right)\left(\sqrt{e^2+f^2}\right)\left(g^2+h^2\right) = \sqrt{A^2+B^2}$

Squaring both sides

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

60. If $\left(\frac{1+i}{1-i}\right)^m = 1$ then find the least positive integral value of m.

Ans. : Given $\left(\frac{1+i}{1-i}\right)^m = 1$

Now, $\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$ [multiply divide numerator and denominator by 1+i]

$\Rightarrow \left[\frac{(1+i)^2}{1^2-i^2}\right]^m = 1$

$\Rightarrow \left(\frac{1^2+i^2+2i}{1+1}\right)^m = 1$

$\Rightarrow \left(\frac{1-1+2i}{2}\right)^m = 1$

$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$

$\Rightarrow i^m = 1$

We can also write, $i^m = i^{4k}$

On equating the powers,

Thus, m = 4k, Where k is some integer.

$\therefore$ 1 is the least positive integer.

Least positive integral value of m is $4 \times 1 = 4$

61. Express $(5 - 3i)^3$ in the form a + ib.

Ans. : We have, $(5-3i)^3 = 5^3 - 3 \times 5^2 \times (3i) + 3 \times 5(3i)^2 - (3i)^3$ $[(a-b)^3 = a^3 - 3a^2b + 3b^2a - b^3]$

= 125 - 225i - 135 + 27i = -10 - 198i

62. Represent the complex number $z = 1 + i\sqrt{3}$ in the polar form.

Ans. : We have, $z = 1 + i\sqrt{3}$

Let $1 + i\sqrt{3} = r(\cos\theta + i\sin\theta)$ ...(i)

On equating real and imaginary parts both sides, we get

$r\cos\theta = 1$ and $r\sin\theta = \sqrt{3}$ ...(ii)

On squaring and adding Eqs. (i) and (ii), we get

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 3$

$\Rightarrow r^2 = 4$

$\Rightarrow r = 2$

$\therefore \cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$

Since, both $\cos\theta$ and $\sin\theta$ are positive.

So, θ lies in first quadrant.

$\therefore \theta = \frac{\pi}{3}$

On putting r = 2 and $\theta = \frac{\pi}{3}$ in Eq. (i), we get

polar form of z = $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

63. Convert the complex number $\frac{-16}{1+\sqrt{3}i}$ into polar form.

Ans. : Given complex number $\frac{-16}{1+\sqrt{3}i}$ convert the complex number in x +iy form

$= \frac{-16}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i}$

$= \frac{-16(1-\sqrt{3}i)}{1-(\sqrt{3}i)^2} = \frac{-16(1-\sqrt{3}i)}{1+3}$

$= -4(1-\sqrt{3}i) = -4+4\sqrt{3}i$

Let -4 = $r\cos\theta, 4\sqrt{3} = r\sin\theta$

By squaring and adding, we get

16 + 48 = r^2 $(\cos^2\theta + \sin^2\theta)$

which gives r^2 = 64, i.e., r = 8

Hence $\cos\theta = -\frac{1}{2}, \sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Thus, the required polar form is r(cosθ + i sinθ) = $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

64. Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$

Ans. : Here, a= $\sqrt{5}$ = c and b = 1

the discriminant of the equation is

$1^2 - 4\times\sqrt{5}\times\sqrt{5} = 1-20 = -19$

Therefore, the solutions are

= $\frac{-1\pm\sqrt{-19}}{2\sqrt{5}} = \frac{-1\pm\sqrt{19}i}{2\sqrt{5}}$

65. Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

Ans. : We have $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

$= \frac{6+9i-4i+6}{2-i+4i+2} = \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$

$= \frac{48-36i+20i+15}{16+9} = \frac{63-16i}{25} = \frac{63}{25} - \frac{16}{25}i$

Therefore, conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ is $\frac{63}{25} + \frac{16}{25}i$ [If z = x + iy then then conjugate is x - iy]

66. Evaluate the following:

i^{528}

Ans. : We know that

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

In order to find i^n Where n > 4, we divide n by 4 to get quotient p and remainder q,

So that $n = 4p+q,\ 0 \le q < 4$

Then $i^n = i^{4p+q}$

$= i^{4p} \times i^q$

$= (i^4)^p \times i^q$

$= i^p \times i^q$

$= i^q\ [\therefore\ 1^{p-1}]$

Hence $i^n = i^q,\ \text{where } 0 \le q < 4$

$\therefore\ i^{528} = i^{4 \times 132}$

$= (i^4)^{132}$

$= 1^{132}$

$= 1$

$\therefore\ (i^{528}) = 1$

67. Find the square root of the following complex numbers:

$4i$

Ans. : Let $z = 4i$

Then $|z| = |4i|$

$= |4i|$ ($\because |z_1 z_2| = |z_1| \times |z_2|$)

$= 4$ ($\because |i| = 1$)

$\therefore \sqrt{4i} = \pm \left\{ \sqrt{\frac{4+0}{2}} + i\sqrt{\frac{4-0}{2}} \right\}$ ($\because y > 0$)

$= \pm \{\sqrt{2} + i\sqrt{2}\}$

$= \pm \sqrt{2}(1 + i)$

68. Evaluate the following:

$i^{30} + i^{40} + i^{60}$

Ans. : We know that

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

In order to find i^n Where n > 4, we divide n by 4 to get quotient p and remainder q,

So that $n = 4p + q,\ 0 \le q < 4$

Then $i^n = i^{4p+q}$

$= i^{4p} \times i^q$

$= (i^4)^p \times i^q$

$= i^p \times i^q$

$= i^q\ [\therefore\ 1^{p-1}]$

Hence $i^n = i^q,$ where $0 \le q < 4$

$\therefore\ i^{30} + i^{40} + i^{60} = i^{4 \times 7} \times i^2 + i^{4 \times 10} + i^{4 \times 15}$

$= 1 \times i^2 + 1 + 1$

$= -1 + 1 + 1$

$= 1$

$i^{30} + i^{40} + i^{60} 1$

69. Evaluate the following:

$i^{49} + i^{68} + i^{89} + i^{110}$

Ans. : We know that

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

In order to find i^n Where n > 4, we divide n by 4 to get quotient p and remainder q,

So that $n = 4p + q,\ 0 \le q < 4$

Then $i^n = i^{4p+q}$

$= i^{4p} \times i^q$

$= (i^4)^p \times i^q$

$= i^p \times i^q$

$= i^q \; [\therefore \; 1^{p-1}]$

Hence $i^n = i^q$, where $0 \le q < 4$

$i^{49} + i^{68} + i^{89} + i^{110} = i^{4 \times 12} \times i^1 + i^{4 \times 17} + i^{4 \times 22} \times i^1 + i^{4 \times 27} \times i^2$

$= 1 \times i + 1 + 1 \times i + 1 \times i^2$

$= i + 1 + i - 1$

$= 2i$

$\therefore i^{49} + i^{68} + i^{89} + i^{110} = 2i$

70. Express the following complex numbers in the standard form a + ib:

$(\frac{1}{1-4i} - \frac{2}{1+i})(\frac{3-4i}{5+i})$

Ans. : $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \frac{(1+i-2(1-4i))}{(1-4i)(1+i)} \times \frac{3-4i}{5+i}$

$= \frac{(1+i-2+8i)}{1(1+i)-4i(1+i)} \times \frac{3-4i}{5+i}$

$= \frac{(-1+9i)}{(1+i-4i+4)} \times \frac{3-4i}{5+i}$

$= \frac{-1(3-4i)+9i(3-4i)}{(5-3i)(5+i)}$

$= \frac{-3+4i+27i+36}{5(5+i)-3i(5+i)}$

$= \frac{33+31i}{25+5i-15i+3}$

$= \frac{33+31i}{28-10i}$

$= \frac{(33+31i)}{28-10i} \times \frac{(28+10i)}{28+10i}$

$= \frac{33 \times 28 + 33 \times 10i + 31i \times 28 + 31i \times 10i}{28^2 + 10^2}$

$= \frac{924+330i+868i-310}{784+100}$

$= \frac{614+1198i}{884}$

$= \frac{614}{884} + \frac{1198}{884}i$

$= \frac{307}{442} + \frac{599}{442}i$

$\therefore \left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \frac{307}{442} + \frac{599}{442}i$

71. If Z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then find the value of $|z_1 + z_2 + z_3|$.

Ans. : $|z_1 + z_2 + z_3| = \left|\frac{z_1\bar{z}_1}{z_1} + \frac{z_2\bar{z}_2}{z_2} + \frac{z_3\bar{z}_3}{z_3}\right|$

$= \left|\frac{|z_1|^2}{z_1} + \frac{|z_2|^2}{z_2} + \frac{|z_3|^2}{z_3}\right|$

$= \frac{\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|}{\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|} \ldots [\because |z_1| = |z_2| = |z_3| = 1]$

$= \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$

$= 1$

72. Express the following complex numbers in the standard form a + ib:

$\frac{(1-i)^3}{1-i^3}$

Ans. : $\frac{(1-i)^3}{1-i^3} = \frac{1^3 - i^3 - 3 \times 1 \times i(1+i)}{1-(-i)}$ $\left[\begin{array}{c} \because\ (a-b)^3 = a^3 - b^3 - 3ab(a+b) \\ \text{and } i^3 = -i \end{array} \right]$

$= \frac{1-(-i)-3i(1-i)}{1+i}$

$= \frac{1+i-3i-3}{1+i}$

$= \frac{-2-2i}{1+i}$

$= \frac{-2(1+i)}{1+i}$

$= -2$

$= -2 + 0i$

$\therefore\ \frac{(1-i)^3}{1-i^3} = -2 + 0i$

73. Find the number of solutions of $z^2 + |z|^2 = 0$.

Ans. : Let $z = x + iy$

$z^2 = (x+iy)^2 = x^2 - y^2 + 2xyi$

$|z|^2 = z\bar{z} = (x+iy)(x-iy) = x^2 + y^2$

$z^2 + |z|^2 = 0$

$x^2 - y^2 + 2xyi + x^2 + y^2 = 0$

$2x^2 + 2xyi = 0$

$\Rightarrow 2x^2 = 0$ and $2xy = 0$

$\Rightarrow x = 0$ and $y \in R$

$\therefore z = 0 + iy$ where $y \in R$

74. Show the following quadratic equation by factorization method:

$27x^2 - 10x + 1 = 0$

Ans. : $27x^2 - 10x + 1 = 0$

We will apply discriminant rule,

$x = \frac{-b \pm \sqrt{D}}{2a}$ ... (A)

Where $D = b^2 - 4ac$

$= (-10)^2 - 4.27.1$

$= 100 - 108$

$= -4$

From (A)

$x = \frac{-(-10) \pm \sqrt{-8}}{54}$

$= \frac{10 \pm 2\sqrt{2}i}{54}$

$\frac{5 \pm \sqrt{2}i}{27}$

$\therefore x = \frac{5}{27} + \frac{\sqrt{2}i}{27}, \frac{5}{27} - \frac{\sqrt{2}}{27}i$

75. Show the following quadratic equation:

$x^2 - (2 + i)\,x - (1 - 7i) = 0$

Ans. : $x^2 - (2 + i)\,x - (1 - 7i) = 0$

$\Rightarrow x^2 - (2 + i)\,x - (1 - 7i) = 0$

$\Rightarrow x^2 - (3 - i)\,x + (1 - 2i)\,x - (1 - 7i) = 0$

$\Rightarrow$ x (x - (3 - i)) + (1 - 2i) (x - (3 - i)) = 0
$\Rightarrow$ [x + (1 - 2i)] [x - (3 - i)] = 0
$\Rightarrow$ x = -1 + 2i, 3 - i

76. Show the following quadratic equation:

$x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}\, i = 0$

Ans. : $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}\, i = 0$

$\Rightarrow x^2 - 3\sqrt{2}x - 2ix + \sqrt{2}i = 0$

$\Rightarrow x(x - 3\sqrt{2}) - 2i(x - 3\sqrt{2}) = 0$

$\Rightarrow (x - 2i)(x - 3\sqrt{2}) = 0$

$\Rightarrow x = 2i \text{ or } 3\sqrt{2}$

77. Show the following quadratic equation:

$(2 + i)\, x^2 - (5 - i)\, x + 2\, (1 - i) = 0$

Ans. : $(2 + i)\, x^2 - (5 - i)\, x + 2\, (1 - i) = 0$

$\Rightarrow (2 + i)\, x^2 - 2x - (3 - i)\, x + 2\, (1 - i) = 0$
$\Rightarrow x\, [2 + ix - 2] - (1 - i)\, [(2 + i)\, x - 2] = 0$
$\Rightarrow [x - (1 - i)]\, [(2 + i)\, x - 2] = 0$
Either $[x - (1 - i)] = 0$ or $[(2 + i)\, x - 2] = 0$

$\Rightarrow x = 1 - i \;\text{ or } x = \frac{2}{2+i}$

$\Rightarrow x = 1 - i \text{ or } x = \frac{2 \times 2 - i}{(2+i)(2-i)}$

$x = \frac{4 - 2i}{4 + 1} = \frac{4}{5} - \frac{2}{5}i$

Thus

$x = 1 - i, \frac{4}{5} - \frac{2}{5}i$

78. Show the following quadratic equation by factorization method:

$x^2 - (2\sqrt{3} + 3i))x + 6\sqrt{3}\, i = 0$

Ans. : $x^2 - (2\sqrt{3} + 3i))x + 6\sqrt{3}\, i = 0$

$\Rightarrow x^2 - 2\sqrt{3x} - 3ix + 6\sqrt{3}i = 0$

$\Rightarrow x(x - 2\sqrt{3}) - 3i(x - 2\sqrt{3}) = 0$

$\Rightarrow (x - 3i)(x - 2\sqrt{3}) = 0$

$\Rightarrow x = 3, 2\sqrt{3}$

79. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$, then find (x, y).

Ans. : $x + iy = \left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$

$= \left(\frac{(1+i)^2}{1-i^2}\right) - \left(\frac{(1-i)^2}{1-i^2}\right)^3$

$= \left(\frac{1+2i+i^2}{1+1}\right)^3 - \left(\frac{1-2i+i^2}{1+1}\right)^3$

$= \left(\frac{2i}{2}\right)^3 - \left(\frac{-2i}{2}\right)^3$

$= i^3 - (-i)^3$

$= 2i^3 = 0 - 2i$

$x = 0$ and $y = -2$

80. Find the value of the following :

$(1 + i)^8 + (1 - i)^8$

Ans. :

$$(1+i)^8 + (1-i)^8 = \left[(1+i)^2\right]^4 + \left[(1-i)^2\right]^4$$
$$= \left[1 + 2i + i^2\right]^4 + \left[1 - 2i + i^2\right]^4$$
$$= [1 + 2i - 1]^4 + [1 - 2i - 1]^4$$
$$= [2i]^4 + [-2i]^4 = 16i^4 + 16i^4$$
$$= 16 \times 1 + 16 \times 1 \quad \because i^4 = 1$$
$$= 16 + 16 = 32$$

*** Given section consists of questions of 3 marks each.** **[48]**

81. Evaluate $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$

$$= \left[(-1)^9 + \frac{1}{i^{24}.i}\right]^3 = \left[-1 + \frac{1}{\left(i^2\right)^{12}.i}\right]^3$$

$$= \left[-1 + \frac{1}{(-1)^{12}.i}\right]^3 = \left[-1 + \frac{1}{1 \times i}\right]^3$$

Ans. : $= \left[-1 + \frac{1}{i}\right]^3 = [-1 - i]^3$

$$= -(1+i)^3 = -\left[1 + i^3 + 3 \times 1 \times i(1+i)\right]$$

$$= -[1 - i + 3i(1+i)] = -\left[1 - i + 3i + 3i^2\right]$$

$$= -[1 - i + 3i - 3] = -[-2 + 2i]$$

$$= 2 - 2i$$

82. Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form.

Ans. : We have, $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$

$$= \left[\frac{1+i-2(1-4i)}{(1-4i)(1+i)}\right]\left(\frac{3-4i}{5+i}\right)$$

$$= \left(\frac{1+i-2+8i}{1+i-4i-4i^2}\right)\left(\frac{3-4i}{5+i}\right)$$

$$= \left(\frac{-1+9i}{1-3i+4}\right)\left(\frac{3-4i}{5+i}\right) \quad [\because i^2 = -1]$$

$$= \left(\frac{-1+9i}{5-3i}\right)\left(\frac{3-4i}{5+i}\right)$$

$$= \frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}$$

$$= \frac{-3+31i+36}{25-10i+3} = \frac{33+31i}{28-10i}$$

$$= \frac{(33+31i)}{(28-10i)} \times \frac{(28+10i)}{(28+10i)}$$

[multiplying numerator and denominator by 28 + 10i]

$$= \frac{924+868i+330i+310i^2}{784-100i^2}$$

$= \frac{924 + 1198i - 310}{784 + 100}$

$= \frac{614 + 1198i}{884} = \frac{307}{442} + \frac{599}{442}i$

83. If $x - iy = \sqrt{\frac{a - ib}{c - id}}$ prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$

Ans. : Here $x - iy = \sqrt{\frac{a - ib}{c - id}}$

Squaring both sides, we get

$(x - iy)^2 = \frac{a - ib}{c - id}$

$\Rightarrow \left|(x - iy)^2\right| = \left|\frac{a - ib}{c - id}\right| \Rightarrow |(x - iy)||x - iy| = \left|\frac{a - ib}{c - id}\right|$

$\Rightarrow \left(\sqrt{x^2 + y^2}\right)\left(\sqrt{x^2 + y^2}\right) = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}} \Rightarrow (x^2 + y^2) = \sqrt{\frac{a^2 + b^2}{c^2 + d^2}}$

Squaring both sides

$(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$

84. Convert in the polar form: $\frac{1 + 7i}{(2 - i)^2}$

Ans. : $\frac{1 + 7i}{(2 - i)^2} = \frac{1 + 7i}{4 + i^2 - 4i} = \frac{1 + 7i}{3 - 4i} \times \frac{3 + 4i}{3 + 4i}$

$= \frac{3 + 4i + 21i + 28i^2}{9 - 16i^2}$

$= \frac{-25 + 25i}{25} = -1 + i$

Let $z = -1 + i = r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = -1$ and $r\sin\theta = 1$ .... (i)

Squaring both sides of (i) and adding

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1 \Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

Since $\sin\theta$ is positive and $\cos\theta$ is negative.

$\therefore \theta$ lies in second quadrant,

$\therefore \theta = \left(\pi - \frac{\pi}{4}\right) = \frac{3\pi}{4}$

Hence polar form of z is $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

85. If a + ib $= \frac{(x + i)^2}{2x^2 + 1}$, prove that $a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$.

Ans. : Here a + ib $= \frac{(x + i)^2}{2x^2 + 1} = \frac{x^2 + i^2 + 2ix}{2x^2 + 1} = \frac{x^2 - 1}{2x^2 + 1} + i\frac{2x}{2x^2 + 1}$

Comparing both sides, we have

$a = \frac{x^2 - 1}{2x^2 + 1}$ amd $b = \frac{2x}{2x^2 + 1}$

$\therefore a^2 + b^2 = \left(\frac{x^2 - 1}{2x^2 + 1}\right)^2 + \left(\frac{2x}{2x^2 + 1}\right)^2$

$= \frac{(x^2 - 1)^2}{(2x^2 + 1)^2} + \frac{(2x)^2}{(2x^2 + 1)^2}$

$= \frac{(x^2-1)^2+(2x)^2}{(2x^2+1)^2} = \frac{x^4+1-2x^2+4x^2}{(2x^2+1)^2}$

$= \frac{x^4+1+2x^2}{(2x^2+1)^2} = \frac{(x^2+1)^2}{(2x^2+1)^2}$

86. If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

Ans. : $(x + iy)^3 = u + iv$

$\Rightarrow x^3 + i^3y^3 + 3x^2yi + 3xy^2i^2 = u + iv$

$\Rightarrow (x^3 - 3xy^2) + (3x^2y - y^3)i = u + iv$

Comparing both sides

$u = x(x^2 - 3y^2)$ and $v = y(3x^2 - y^2)$

Now $\frac{u}{x} + \frac{v}{y} = \frac{x(x^2-3y^2)}{x} + \frac{y(3x^2-y^2)}{y}$

$= x^2 - 3y^2 + 3x^2 - y^2 = 4x^2 - 4y^2 = 4(x^2 - y^2)$

87. If α and β are different complex numbers with $|\beta| = 1$ then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

Ans. : Now $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = \left[\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right]\left[\overline{\frac{\beta-\alpha}{1-\bar{\alpha}\beta}}\right] \left[\because |z|^2 = z\bar{z}\right]$

$= \left[\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right]\left[\frac{\bar{\beta}-\bar{\alpha}}{1-\alpha\bar{\beta}}\right]$

$= \frac{\beta\bar{\beta}-\beta\bar{\alpha}-\alpha\bar{\beta}+\alpha\bar{\alpha}}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+\bar{\alpha}\alpha\beta\bar{\beta}} = \frac{|\beta|^2-\bar{\alpha}\beta-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2|\beta|^2}$

$= \frac{1-\bar{\alpha}\beta-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2} = 1$

$\therefore \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$

88. Express $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$ in the form of a + ib.

Ans. : Let $z = \frac{5+\sqrt{2}i}{1-\sqrt{2}i} = \frac{5+\sqrt{2}i}{1-\sqrt{2}i} \times \frac{1+\sqrt{2}i}{1+\sqrt{2}i}$

[multiplying numerator and denominator by $1 + \sqrt{2}i$]

$= \frac{5+5\sqrt{2}i+\sqrt{2}i-2}{1-(\sqrt{2}i)^2}$

$= \frac{3+6\sqrt{2}i}{1+2}$

$= \frac{3(1+2\sqrt{2}i)}{3}$

$= 1 + 2\sqrt{2}\,i$

89. If $x + iy = \frac{a+ib}{a-ib}$, prove that $x^2 + y^2 = 1$

Ans. : We have $x + iy = \frac{(a+ib)}{(a-ib)}$

$\Rightarrow |x + iy| = \left|\frac{(a+ib)}{(a-ib)}\right|$

Squaring Both the sides,

=> $|x + iy|^2 = \frac{|(a+ib)|^2}{|(a-ib)|^2}$

=> $x^2 + y^2 = \frac{a^2+b^2}{a^2+b^2} = 1$

90. Write the complex number $z = \dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ in the polar form.

Ans. : We have, $z = \dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$

Let, $-1 + i = r(\cos\theta + i\sin\theta)$

$\Rightarrow$ $r\cos\theta = -1$ ...(i)

and $r\sin\theta = 1$ ...(ii)

On squaring and adding Eqs. (i) and (ii), we get

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$

$\Rightarrow$ $r^2 = 2$

$\therefore$ $r = \sqrt{2}$ [taking positive square root]

On putting the value of r in Eqs. (i) and (ii), we get

$\cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

Since, $\sin\theta$ is positive and $\cos\theta$ is negative.

So, θ lies in II quadrant.

$\therefore$ $\theta = \left(\pi - \frac{\pi}{4}\right) = \frac{3\pi}{4}$

$\Rightarrow$ $i - 1 = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$\therefore$ $z = \dfrac{i-1}{\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)} = \dfrac{\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)}{\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$

$= \dfrac{\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)}{\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)} \times \dfrac{\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)}{\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)}$ [multiplying numerator and denominator by $\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$]

$= \dfrac{\sqrt{2}\left[\left(\cos\frac{3\pi}{4}\cdot\cos\frac{\pi}{3} + \sin\frac{3\pi}{4}\cdot\sin\frac{\pi}{3}\right) + i\left(\sin\frac{3\pi}{4}\cdot\cos\frac{\pi}{3} - \cos\frac{3\pi}{4}\cdot\sin\frac{\pi}{3}\right)\right]}{\left(\cos^2\frac{\pi}{3} + \sin^2\frac{\pi}{3}\right)}$

$= \dfrac{\sqrt{2}\left[\cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)\right]}{1}$

$= \sqrt{2}\left[\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right]$

91. $(1 + i)^6 + (1 - i)^3$

Ans. : $(1 + i)^6 + (1 - i)^3 = \left[(1 + i)^2\right]^3 + (1 - i)^3$

$= (1 + i^2 + 2i)^3 + (1 - 3i + 3i^2 - i^3)$

$= (1 - 1 + 2i)^3 + (1 - 3i - 3 + i)$

$= 8i^3 - 2 - 2i$

$= -2 - 10i$

92. Find the values of the following expressions:

$i^{30} + i^{80} + i^{120}$

Ans. : $i^{30} + i^{80} + i^{120} = i^{4\times 7} \times i^2 + i^{4\times 20} + i^{4\times 30}$

$= 1 \times i^2 + 1 + 1$

$= -1 + 1 + 1$

$= 1$

$\therefore i^{30} + i^{80} + i^{120} = 1$

93. Show the following quadratic equation by factorization method:

$\sqrt{2}x^2 + x + \sqrt{2} = 0$

Ans. : $\sqrt{2}x^2 + x + \sqrt{2} = 0$

We will apply discriminant rule,

$x = \frac{-b \pm \sqrt{D}}{2a}$ ...(A)

Where D = b^2 - 4ac

$= 1^2 - 4.\sqrt{2}.\sqrt{2}$

= 1 - 8

= -7

From (A)

$x = \frac{-1 \pm \sqrt{-7}}{2.\sqrt{2}}$

$= \frac{-1 \pm \sqrt{7}\,i}{2\sqrt{2}}$

$\therefore x = \frac{-1 \pm \sqrt{7}\,i}{2\sqrt{2}}$

94. Show the following quadratic equation:

ix^2 - x + 12i = 0

Ans. : We will apply discriminate rule on $ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Now,

ix^2 - x + 12i = 0

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(i)(12i)}}{2i}$

$= \frac{1 \pm \sqrt{1 + 48}}{2i}$

$= \frac{1 \pm \sqrt{49}}{2i}$

$= \frac{1 \pm 7}{2i}$

$= \frac{8}{2i}, \frac{-6}{2i}$

$= \frac{4}{i}, -\frac{3}{i}$

$= -4i, 3i$

95. If $\frac{(1+i)^2}{2-i} = x + iy$, then find the value of x + y.

Ans. : Given that, $\frac{(1+i)^2}{2-i} = x + iy$

$\Rightarrow \frac{1+i^2+2i}{2-i} = x + iy$

$\Rightarrow \frac{1-1+2i}{2-i} = x + iy$

$\Rightarrow \frac{2i}{2-i} = x + iy$

$\Rightarrow \frac{2i(2+i)}{(2-i)(2+i)} = x + iy$

$\Rightarrow \frac{4i+2i^2}{4-i^2} = x + iy$

$\Rightarrow \frac{4i-2}{4+1} = x + iy\ [\because\ i^2 = -1]$

$\Rightarrow \frac{-2+4i}{5} = x + y$

$= \frac{-2}{5} + \frac{4}{5}i = x + iy$

Comparing the real and imaginary parts, we get

$x = \frac{-2}{5}$ and $y = \frac{-4}{5}$

Hence, $x+y = \frac{-2}{5} + \frac{4}{5} = \frac{2}{5}$

96. If $\frac{(a^2+1)^2}{2a-i} = x + iy$, what is the value of $x^2 + y^2$?

Ans. : Given that, $\frac{(a^2+1)^2}{2a-i} = x + iy \ldots$ (i)

Taking conjugate on both sides,

$\frac{(a^2+1)^2}{2a+i} = x - iy \ldots$ (ii)

Multiplying eq. (i) and (ii) we have

$\frac{(a^2+1)^2(a^2+1)^2}{(2a-i)(2a+i)} = x^2 + y^2$

$\Rightarrow \frac{(a^2+1)^4}{4a^2-i^2} = x^2 + y^2$

$\Rightarrow \frac{(a^2+1)^4}{4a^2-i^2} = x^2 + y^2$

Hence, the value of $x^2 + y^2 = \frac{(a^2+1)^4}{4a^2+1}$

* **Given section consists of questions of 5 marks each.** **[55]**

97. Express the following complex numbers in the form $r(\cos\theta + i\sin\theta)$:
$1 - \sin\alpha + i\cos\alpha$

Ans. : Let $z = (1 - \sin\alpha) + i\cos\alpha$

Since sine and cosine are periodic functions with periodic with peroid 2π.

So, let us take α lying in the interval $[0, 2\pi]$

Now, $z = (1 - \sin\alpha) + i\cos\alpha$

$\Rightarrow |z| = \sqrt{(1-\sin\alpha)^2 + \cos^2\alpha} = \sqrt{2 - 2\sin\alpha} = \sqrt{2}\sqrt{1 - \sin\alpha}$

$\Rightarrow |z| = \sqrt{2}\sqrt{\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)^2} = \sqrt{2}\left|\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right|$

Let β be acute angle given by $\tan\beta = \frac{|\mathrm{Im}(z)|}{|\mathrm{Re}(z)|}$.

$\tan\beta = \frac{|\cos\alpha|}{|1-\sin\alpha|} = \frac{|\cos\alpha|}{|1-\sin\alpha|} = \left|\frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)}\right| = \left|\frac{\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}}\right|$

$\Rightarrow \tan\beta = \left|\frac{1+\tan\frac{\alpha}{2}}{1-\tan\frac{\alpha}{2}}\right| = \left|\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right|$

Following cases arise:

Case I: when $0 \le \alpha < \frac{\pi}{2}$

$\cos\frac{\alpha}{2} > \sin\frac{\alpha}{2}$ and $\frac{\pi}{4} + \frac{\alpha}{2} \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\therefore \arg(z) = \frac{\pi}{2} + \frac{\alpha}{2}$

So polar form of z is

$\sqrt{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)\left(\cos\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) + i\sin\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right)$

Case II: when $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

$\cos\frac{\alpha}{2} < \sin\frac{\alpha}{2}$ and $\frac{\pi}{4} + \frac{\alpha}{2} \in \left(\frac{\pi}{2}, \pi\right)$

$\therefore |z| = \sqrt{2}\left|\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right| = \sqrt{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)$

and,

$\tan\beta = \left|\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right| = -\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) = \tan\left\{\pi\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right\} = \tan\left(\frac{3\pi}{4} - \frac{\alpha}{2}\right)$

$\Rightarrow \beta = \frac{3\pi}{4} - \frac{\alpha}{2}$

Since $1 - \sin\alpha > 0$ and $\cos\alpha < 0$.

Clearly, z lies in the fourth quadrant.

$\therefore \arg(z) = -\beta = \frac{\alpha}{2} - \frac{3\pi}{4}$

So polar form of z is $\sqrt{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)\left(\cos\left(\frac{\alpha}{2} - \frac{3\pi}{4}\right) + i\sin\left(\frac{\alpha}{2} - \frac{3\pi}{4}\right)\right)$

Case III: when $\frac{3\pi}{2} < \alpha < 2\pi$

$\cos\frac{\alpha}{2} < \sin\frac{\alpha}{2}$ and $\frac{\pi}{4} + \frac{\alpha}{2} \in \left(\pi, \frac{5\pi}{4}\right)$

$\therefore |z| = \sqrt{2}\left|\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right| = -\sqrt{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)$

and,

$\tan\beta = \left|\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right| = \tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) = -\tan\left\{\pi - \left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right\} = \tan\left(\frac{\alpha}{2} - \frac{3\pi}{4}\right)$

$\Rightarrow \beta = \frac{\alpha}{2} - \frac{3\pi}{4}$

Clearly, $\mathrm{Re}(z) < 0$ and $\mathrm{Im}(z) > 0$.

$\therefore \arg(z) = \beta = \frac{\alpha}{2} - \frac{3\pi}{4}$

So polar form of z is $-\sqrt{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)\left(\cos\left(\frac{\alpha}{2} - \frac{3\pi}{4}\right) + i\sin\left(\frac{\alpha}{2} - \frac{3\pi}{4}\right)\right)$.

98. Find the least positive integral value of n for which $\left(\frac{1+i}{1-i}\right)^n$ is real.

Ans. : For n = 1, we have,

$\left(\frac{1+i}{1-i}\right)^1 = \frac{1+i}{1-i}$

$= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{(1+i)^2}{1^2+1^2}$

$= \frac{1^2+i^2+2\times1\times i}{2}$

$= \frac{2i}{2}$ $(\therefore\ i^2 = -1)$

$= i,$ which is not real

For n = 2, we have

$\left(\frac{1+i}{1-i}\right)^2 = i^2$ $\left(\because\ \frac{1+i}{1-i} = 1 \text{ from above}\right)$

$= -1,$ which is real

Hence the least positive integral value of n is 2.

99. Evaluate the following:

$2x^3 + 2x^2 - 7x + 72,$ when $x = \frac{3-5i}{2}$

Ans. : $x = \frac{3-5i}{2}$

$\Rightarrow 2x = 3 - 5i$

$\Rightarrow 2x - 3 = -5i$

$\Rightarrow (2x-3)^2 = (-5i)^2$

$\Rightarrow 4x^2 + 9 - 12x = -25$

$\Rightarrow 4x^2 - 12x + 34 = 0$

$\Rightarrow 2(2x^2 - 6x + 17) = 0$

$\Rightarrow 2x^2 - 6x + 17 = 0 \ldots (i)$

$\therefore 2x^3 + 2x^2 - 7x + 72$

$= x(2x^2 - 6x + 17) + 6x^2 - 17x + 2x^2 - 7x + 72$ (Adding and subtracting $6x^2$ and 17x)

$x \times 0 + 8x^2 - 24x + 72$ (Using(i))

$= 4(2x^2 - 6x + 17) + 4$

$= 4 \times 0 + 4$ (Using(i))

$= 4$

100. $x^4 + 4x^3 + 6x^2 + 4x + 9,$ when $x = -1 + i\sqrt{2}$

Ans. : We have

$x = -1 + i\sqrt{2}$

$\Rightarrow x + 1 + i\sqrt{2}$

$\Rightarrow (x+1)^2 = (i\sqrt{2})^2$ (squaring both sides)

$\Rightarrow x^2 + 1 + 2x = -2$

$\Rightarrow x^2 + 2x + 3 = 0 \ldots (i)$

Now,

$x^4 + 4x^3 + 6x^2 + 4x + 9$

$= x^2(x^2 + 2x + 3) + 2x^3 + 3x^2 + 4x + 9$

$= x^2 \times 0 + 2x(x^2 + 2x + 3) - x^2 - 2x + 9$ (using(i))

$= 2x \times 0 - (x^2 + 2x + 3) + 3 + 9$ (using(i) and adding and subtracting 3)

$= -0 + 3 + 9$ (using(i))

$= 12$

101. Express the following complex numbers in the form $r(\cos\theta + i\sin\theta)$:

$\tan\alpha - i$

Ans. : Let $z = \tan\alpha - i$

$\tan\alpha$ is periodic function with period π

So, let us take α lying in the interval $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$.

Case I: when $\alpha \in \left[0, \frac{\pi}{2}\right)$

$|z| = \sqrt{\tan^2\alpha + 1} = \sqrt{\sec^2\alpha} = |\sec\alpha| = \sec\alpha$

Let β be acute angle given by $\tan\beta = \frac{|\mathrm{Im}(z)|}{|\mathrm{Re}(z)|}$.

$\tan\beta = \frac{1}{|\tan\alpha|} = |\cot\alpha| = \cot\alpha = \tan\left(\frac{\pi}{2} - \alpha\right)$

$\Rightarrow \beta = \frac{\pi}{2} - \alpha$

As z represented by a point in first quadrant.

$\therefore \arg(z) = \beta = \alpha - \frac{\pi}{2}$.

So polar form of z is $\sec\alpha\left(\cos\left(\alpha - \frac{\pi}{2}\right) + i\sin\left(\alpha - \frac{\pi}{2}\right)\right)$

Case II: when $\alpha \in \left(\frac{\pi}{2}, \pi\right]$

$|z| = \sqrt{1 + \tan^2\alpha + 1} = \sqrt{\sec^2\alpha} = |\sec\alpha| = -\sec\alpha$

Let β be acute angle given by $\tan\beta = \frac{|\mathrm{Im}(z)|}{|\mathrm{Re}(z)|}$.

$\tan\beta = |\tan\alpha| = -\tan\alpha = \tan(\pi - \alpha)$

$\Rightarrow \beta = \alpha - \frac{\pi}{2}$

As z represented by a point in fourth quadrant.

$\therefore\ \arg(z) = \pi + \beta = \frac{\pi}{2} + \alpha.$

So polar form of z is $-\sec\alpha\left(\cos\left(\frac{\pi}{2} + \alpha\right) + i\sin\left(\frac{\pi}{2} + \alpha\right)\right)$.

102. $2x^4 + 5x^3 + 7x^2 - x + 41$, when $x = -2 - \sqrt{3}i$

Ans. : $x = -2 - \sqrt{3}i$

$x^2 = (-2 - \sqrt{3}i)^2 = 4 + 4\sqrt{3}i + 3i^2 = 1 + 4\sqrt{3}i$

$x^3 = (1 + 4\sqrt{3}i)(-2 - \sqrt{3}i) = -2 - 8\sqrt{3}i - \sqrt{3}i - 12i^2 = 10 + 9\sqrt{3}i$

$x^4 = (1 - 4\sqrt{3}i)^2 = 1 + 8\sqrt{3}i + 48i^2 = -47 + 8\sqrt{3}i$

$2x^4 + 5x^3 + 7x^2 - x + 41$

$= 2(-47) + 8\sqrt{3}i) + 5(10 - 9\sqrt{3}i) + 7(1 + 4\sqrt{3}i) - (-2 - \sqrt{3}i) + 41$

$= -94 + 16\sqrt{3}i + 50 - 45\sqrt{3}i + 7 + 28\sqrt{3}i + 2 + \sqrt{3}i + 41$

$= (-94 + 50 + 7 + 2 + 41) + (16\sqrt{3}i - 45\sqrt{3}i + 28\sqrt{3}i + \sqrt{3}i)$

$= 6 + 0$

$= 6$

103. For a positive integer n, find the value of $(1 - i)^n(1 - \frac{1}{i})^i$.

Ans. : $(1 - i)^n\left(1 - \frac{1}{i}\right)^n$

$= (1 - i)^n\left(1 - \frac{1}{i}\right)^n$

$= \left\{\frac{(1-i)(i-1)}{i}\right\}^n$

$= \left\{\frac{(1-i)(1-i)}{-i}\right\}^n$

$= \left\{\frac{(1-i)^2}{-i}\right\}^n$

$= \left\{\frac{(1-2i-1)}{-i}\right\}^n$

$= \left\{\frac{-2i}{-i}\right\}^n = 2^n$

104. Express the following complex numbers in the form $r(\cos\theta + i\sin\theta)$:
$1 + i\tan\alpha$

Ans. : Let $z = 1 + i\tan\alpha$

$\tan\alpha$ is periodic function with period π

So, let us take α lying in the interval $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$.

Case - 1: when $\alpha \in \left[0, \frac{\pi}{2}\right)$

$|z| = \sqrt{1 + \tan^2\alpha} = \sqrt{\sec^2\alpha} = |\sec\alpha| = \sec\alpha$

Let β be acute angle given by $\tan\beta = \frac{|\mathrm{Im}(z)|}{|\mathrm{Re}(z)|}$.

$\tan\beta = |\tan\alpha| = \tan\alpha$

$\Rightarrow \beta = \alpha$

As z represented by a point in first quadrant.

$\therefore\ \arg(z) = \beta = \alpha$

So polar form of z is $\sec\alpha(\cos\alpha + i\sin\alpha)$

Case - 2: when $\alpha \in \left(\frac{\pi}{2}, \pi\right]$

$|z| = \sqrt{1 + \tan^2\alpha} = \sqrt{\sec^2\alpha} = |\sec\alpha| = -\sec\alpha$

Let β be acute angle given by $\tan\beta = \frac{|\text{Im}(z)|}{|\text{Re}(z)|}$.

$\tan\beta = |\tan\alpha| = -\tan\alpha = \tan(\pi - \alpha)$

$\Rightarrow \beta = \pi - \alpha$

As z represented by a point in fourth quadarnt.

$\therefore \arg(z) = -\beta = \alpha - \pi$

So polar form of z is $-\sec\alpha\big(\cos(\alpha - \pi) + i\sin(\alpha - \pi)\big)$.

105. Evaluate the following:

$x^4 - 4x^3 + 4x^2 + 8x + 44,$ when $x = 3 + 2i$

Ans. : We have,

$x = 3 + 2i$

$\Rightarrow x - 3 = 2i$

$\Rightarrow (x - 3)^2 = (2i)^2$

$\Rightarrow x^2 + 3^2 - 2 \times 3 \times x = -4$

$\Rightarrow x^2 + 9 - 6x + 4 = 0$

$\Rightarrow x^2 - 6x + 13 = 0 \ldots (i)$

Now,

$x^4 - 4x^3 + 4x^2 + 8x + 44$

$= x^2(x^2 - 6x + 13) + 6x^2 - 13x^2 - 4x^3 + 4x^2 + 8x + 44$ (adding and subtracting $6x^3$ and $13x^2$)

$= x^2 \times 0 + 2x^3 - 9x^2 + 8x^2 + 44$ (using (i))

$2x(x^2 - 6x + 13) + 12x^2 - 26x - 9x^2 + 8x + 44$ (adding and subtracting $12x^3$ and $26x^2$)

$2x \times 0 + 3x^2 - 18x + 44$ (using (i))

$= 3(x^2 - 6x + 13) + 5$

$= 3 \times 0 + 5$ (using (i))

$= 5$

106. If $(\frac{1 - i}{1 + i})^{100} = a + ib,$ find (a, b).

Ans. : $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$

$\Rightarrow \left(\frac{(1-i)(1-i)}{(1+i)(1-i)}\right)^{100} = a + ib$ [Rationalizing the denominator]

$\Rightarrow \left(\frac{(1-2i-1)}{(1+1)}\right)^{100} = a + ib$

$\Rightarrow \left(\frac{-2i}{2}\right)^{100} = a + ib$

$\Rightarrow (-i)^{100} = a + ib$

$\Rightarrow 1 = a + ib$

Comparing, we get

$(a, b) = (1, 0)$

107. If $a = \cos\theta + i\sin\theta,$ find the value of $(\frac{1 + a}{1 - a})$

Ans. : Given that, $a = \cos\theta + i\sin\theta$

$\therefore \frac{1+a}{1-a} = \frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta}$

$= \frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta} \times \frac{1-\cos\theta+i\sin\theta}{1-\cos\theta+i\sin\theta}$

$= \frac{1-\cos\theta+i\sin\theta+\cos\theta-\cos^2\theta+i\sin\theta\cos\theta+i\sin\theta-i\sin\theta\cos\theta+i^2\sin^2\theta}{(1-\cos\theta)^2-i^2\sin^2\theta}$

$= \frac{1 + \text{isin } \theta - \cos^2 \theta + \text{isin } \theta - \sin^2 \theta}{1 + \cos^2 \theta - 2\cos \theta + \sin^2 \theta}$

$= \frac{2\text{isin } \theta}{2(1 - \cos \theta)} = \frac{\text{isin } \theta}{1 - \cos \theta}$

$= \frac{2\sin \frac{\theta}{2}\cos \frac{\theta}{2}\text{i}}{2\sin^2 \frac{\theta}{2}}$

$= \cot\frac{\theta}{2}\text{i}$

Hence, $\frac{1 + \text{a}}{1 - \text{a}} = \text{i}\cot\frac{\theta}{2}$

*** Case study based questions** **[8]**

108. A complex number z is pure real if and only if $\bar{z} = z$ and is pure imaginary if and only if $\bar{z} = -z$.

Based on the above information, answer the following questions.

(i) If $(1 + i)z = (1 - i)\bar{z}$, then $-i\bar{z}$ is

(a) $-\bar{z}$ (b) z (c) $\bar{z}$ (d) z^{-1}

(ii) $\overline{Z_1 Z_2}$ is

(a) $\bar{z}_1\bar{z}_2$ (b) $\bar{z}_1 + \bar{z}_2$ (c) $\frac{z_1}{z_2}$ (d) $\frac{1}{z_1 z_2}$

(iii) If x and y are real numbers and the complex number $\frac{(2+i)x - i}{4+i} + \frac{(1-i)y + 2i}{4i}$ is pure real, the relation between x and y is

(a) $8x - 17y = 16$ (b) $8x + 17y = 16$

(c) $17x - 8y = 16$ (d) $17x - 8y = -16$

(iv) If $z = \frac{3 + 2i\sin \theta}{1 - 2i\sin \theta}\left(0 < \theta \le \frac{\pi}{2}\right)$ is pure imaginary, then θ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{4}{6}$ (c) $\frac{6}{3}$ (d) $\frac{\pi}{12}$

(v) If z_1 and z_2 are complex numbers such that $\frac{z_1 - z_2}{|z_1 + z_2|} = 1$

(a) $\frac{z_1}{z_2}$ is pure real (b) $\frac{z_1}{z_2}$ is pure imaginary

(c) z_1 is pure real (d) z_1 and z_2 are pure imaginary

Ans. : $(i) - (b); (i) - (a); (ii) - (a); (iv) - (c); (v) - (b)$

109. We have, $i = \sqrt{-1}$. So, we can write the higher powers of i as follows

(i) $i^2 = -1$

(ii) $i^3 = i^2 \cdot i = (-1) \cdot i = -i$

(iii) $i^4 = (i^2)^2 = (-1)^2 = 1$

(iv) $i^5 = i^{4+1} = i^4 \cdot i = 1 \cdot i = i$

(v) $i^6 = i^{4+2} = i^4 \cdot i^2 = 1 \cdot i^2 = -1$

In order to compute i^n for $n > 4$, write $i^n = i^{4q+r}$ for some $q, r \in N$ and $0 \le r \le 3$. Then, $i^n = i^{4q} \cdot i^r = (i^4)^q \cdot i^r = (1)^q \cdot i^r = i^r$.

In general, for any integer $k, i^{4k} = 1, i^{4k+1} = i, i^{4k+2} = -1$ and

$i^{4k+3} = -i.$

On the basis of above information, answer the following questions.

(i) The value of i^{37} is equal to
(a) i (b) $-i$ (c) 1 (d) -1

(ii) The value of i^{-30} is equal to
(a) i (b) 1 (c) -1 (d) $-i$

(iii) If $z = i^9 + i^{19}$, then z is equal to
(a) $0 + 0i$ (b) $1 + 0i$ (c) $0 + i$ (d) $1 + 2i$

(iv) The value of $\left[i^{19} + \left(\frac{1}{i}\right)^{25}\right]^2$ is equal to
(a) -4 (b) 4 (c) i (d) 1

(v) If $z = i^{-39}$, then simplest form of z is equal to
(a) $1 + 0i$ (b) $0 + i$ (c) $0 + 0i$ (d) $1 + i$

Ans. : (i) - (a); (ii) - (c); (iii) - (a); (iv) - (a); (v) - (b)

----- “The only one who can tell you 'you can't win' is you, and you don't have to listen.” -----